# PRINCIPAL COMPONENTS ANALYSIS OF TRIAXIAL VIBRATION DATA FROM HELICOPTER TRANSMISSIONS \*

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#### **Abstract:**

Research on the nature of the vibration data collected from helicopter transmissions during flight experiments has led to several crucial observations believed to be responsible for the high rates of false alarms and missed detections in aircraft vibration monitoring systems. This work focuses on one such finding, namely, the need to consider additional sources of information about system vibrations. In this light, helicopter transmission vibration data, collected using triaxial accelerometers, were explored in three different directions, analyzed for content, and then combined using Principal Components Analysis (PCA) to analyze changes in directionality. In this paper, the PCA transformation is applied to 176 test conditions/data sets collected from an OH58C helicopter to derive the overall experiment-wide covariance matrix and its principal eigenvectors. The experiment-wide eigenvectors are then projected onto the individual test conditions to evaluate changes and similarities in their directionality based on the various experimental factors. The paper will present the foundations of the proposed approach, addressing the question of whether experiment-wide eigenvectors accurately model the vibration modes in individual test conditions. The results will further determine the value of using directionality and triaxial accelerometers for vibration monitoring and anomaly detection.

# **Keywords:**

Vehicle health monitoring; Helicopter transmission vibration analysis; Gear vibration diagnostics; Principal components analysis; Triaxial vibration measurements.

# **Background and Motivation:**

Detection of anomalies in rotating machinery during flight in high-risk aircraft is a challenging task. Ongoing research at NASA Ames Research Center focuses on collecting in-flight and test-rig vibration data to study the inherent statistical variations in rotating equipment vibrational response. The purpose of this research is

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to identify factors that have the potential of invalidating the overall signal modeling assumptions for vibration monitoring systems, potentially resulting in false alarms and missed detections [4, 12]. The results so far have led to several interesting conclusions about the source of false alarms and missed detections. In this work, one aspect is being explored, namely, the value of additional sources of information provided by the use of multi-axis accelerometers.

Data from triaxial accelerometers have been studied initially to establish that different directions provide information about different dynamic phenomena in the helicopter transmissions and engines [13]. In this work, models of overall healthy and faulty triaxial vibration data are sought to identify changes in the directionality of the data. Principal components analysis (PCA) is used to rotate the three axes to obtain an optimal direction, determined by the principal axis and its angles with the original axes. The following sections first summarize the flight experiments and vibration measurements, along with some previous results using these data. Next, the PCA approach is summarized and an application to a single test condition is presented as an example. Finally, the relationship between the PCA eigenvectors for the whole set of test conditions and PCA eigenvectors for a single test condition is explored using triaxial vibration data. More specifically, the aggregated 176 data sets collected from an OH58C helicopter are used to derive the experiment-wide covariance matrix and its principal eigenvectors. These eigenvectors represent the overall vibrational modes in the data, which are then projected onto individual test conditions to determine their predictive value. It is hoped that the results will contribute to the understanding of the value of triaxial recordings in vibration monitoring, the potential of generating generalized data models for enhanced failure and anomaly detection in aircraft systems, and, the value of using directionality as a metric for vibration monitoring and anomaly detection.

## **Helicopter Flight Experiments:**

The present helicopter flight experiments were conducted by taking vibration recordings during a predetermined set of flight conditions; these constituted fourteen maneuvers. The experiments are based on a latin square design which counter-balances the flight conditions to assure that gross weight and ambient temperature changes do not bias the results [4, 10]. The use of a carefully designed experiment allows for various sources of variation and their interactions to be investigated and quantified in a systematic fashion. In this experimental design, two pilots fly fourteen maneuvers each, and repeat each maneuver three times, in two different sets. The maneuvers are selected based on discussions with the research pilots and are designed to cover a representative set of stable conditions typical of helicopter flight. Each "flight" consists of 22 maneuvers, resulting in 176 files (test conditions) total. Test conditions refer to each combination of maneuver, pilot, training set, and order. The test conditions for the OH58C test flights are detailed in Table 1 for Flights 1-4 [13], which are repeated in Flights 5-8 to generate a second set of vibration data.

# **Triaxial Data Analysis:**

Typically, vibration monitoring is performed using single-axis accelerometers placed radially on the transmission housing [3, 5, 8]. The value in the directionality of the frequency content has been explored in literature by using several single-axis accelerometers mounted in various directions [7]. This work indicated the possibility of different signatures being emphasized in the different directions that were studied. While acceptable for test stands, weight and space limitations prohibit the use of additional accelerometers in actual helicopters. As a result, this work addresses the question of whether vibration measurements using triaxial accelerometers can provide an effective technique to categorize baseline changes due to the statistical

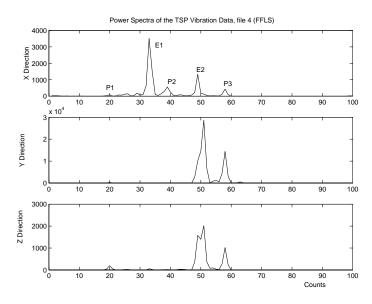
**Table 1:** Experimental design for flights 1-4.

Flight	Pilot	Set	Sequence	Flight	Pilot	Set	Sequence
1	1(d)	1	G, Ground	2	1(d)	1	G, Ground
			H, Hover				H, Hover
			A, FFLS				I, HTL
			B, FFHS				J, HTR
			C, SL				K, CTL
			D, SR				L, CTR
			E, FCLP				M, FCHP
			F, FDLP				N, FDHP
			B, FFHS				J, HTR
			C, SL				K, CTL
			D, SR				L, CTR
			E, FCLP				M, FCHP
			F, FDLP				N, FDHP
			A, FFLS				I, HTL
			C, SL				K, CTL
			D, SR				L, CTR
			E, FCLP				M, FCHP
			F, FDLP				N, FDHP
			A , FFLS				I, HTL
			B , FFHS				J, HTR
			H, Hover				H, Hover
			G, Ground				G, Ground
3	2(h)	1	G, Ground	4	2(h)	1	G, Ground
3	2(h)	1	H, Hover	4	2(h)	1	H, Hover
3	2(h)	1	H , Hover A , FFLS	4	2(h)	1	H, Hover I, HTL
3	2(h)	1	H , Hover A , FFLS B , FFHS	4	2(h)	1	H, Hover I, HTL J, HTR
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL K, CTL
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL D, SR	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL K, CTL L, CTR
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL D, SR E, FCLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL K, CTL L, CTR M, FCHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL D, SR E, FCLP F, FDLP	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL K, CTL L, CTR M, FCHP N, FDHP
3	2(h)	1	H, Hover A, FFLS B, FFHS C, SL D, SR E, FCLP F, FDLP C, SL D, SR E, FCLP F, FDLP A, FFLS C, SL D, SR E, FCLP A, FFLS C, SL D, SR	4	2(h)	1	H, Hover I, HTL J, HTR K, CTL L, CTR M, FCHP N, FDHP K, CTL L, CTR M, FCHP N, FDHP I, HTL K, CTL L, CTR M, FCHP I, HTL L, CTR M, FCHP N, FDHP I, HTL L, CTR

variation in the vibration data, collected from an OH58C helicopter transmission gearbox.

For this set of experiments, the accelerometers were mounted on the bolts around the housing at four locations. A specially-designed HealthWatch-I data collection system collects eight channels of data including vibration data from three single-axis accelerometers and one triaxial accelerometer [4, 13]. The channels were sampled at a rate of 50kHz per channel, for a duration of 34 seconds, corresponding to over 190 revolutions of the output rotor [5].

The frequency content was analyzed for each test condition to determine the differences observed in the three directions of the triaxial accelerometer [13]. In an attempt to isolate frequencies specific to different gears in the transmission, the raw vibration data were averaged using time-synchronous averaging (TSA) techniques [1, 2, 9, 11]. In this paper, results are reported using data computed from the raw triaxial accelerometer data averaged based on one revolution of the pinion gear (referred to as the "TSP data", N = 512 sample points) [13]. The power spectra for the TSP data contain all frequencies that are synchronous with the pinion gear rotation, as shown in Figure 1 for the case of the TSP data set using Flight 1, file 4, maneuver FFLS as an example. The x-axis of the spectra is presented in frequency "counts", which corresponds to the frequency divided by the rotational frequency of the gear of interest, or "shaft order". For example, for the TSP data, a frequency component at bin 19 will correspond to the pinion mesh frequency, equal to the number of teeth ( $N_{teeth} = 19$ ) in the pinion multiplied by its rotational frequency.



**Figure 1:** Power spectra in X, Y, and Z directions, triaxial accelerometer, TSP data. Flight 1, file 4, maneuver FFLS (Forward Flight, Low Speed).

The results of this analysis showed that each of the three directions can be used to monitor different components of the frequency distribution, highlighting a potential benefit of using triaxial accelerometers in addition to single-axis accelerometers [13]. This becomes much more evident in the case of actual flight conditions where different maneuvers can result in an increase or decrease of the vibrational energy in different directions. The results also give additional insight about the directionality of the vibration depending on the gear set under study.

## **PCA-Based Triaxial Analysis Method:**

Triaxial vibration data can either be analyzed separately in each of the three measurement directions, or combined in some mathematical form for analysis. The methodology in this paper performs a Principal Components Analysis (PCA) on the triaxial data to put the three axes of measurement into one "principal axis" with maximum variance [6, 13, 14, 15]. This method of combining the three axes of vibration recordings hierarchically reorganizes the orthogonal variations, while removing the correlation between the physical recording axes. The following presents the foundations of this technique by applying it to empirical data collected during flight. In the following subsections, PCA transformation is first performed on a single test condition, followed by a transformation using the entire set of experiments to derive generalized, experiment-wide eigenvectors. The generalized eigenvectors are then compared with the eigenvectors from individual test conditions to determine whether they can be used to predict the vibrational modes for each test condition. If the answer is yes, then the generalized eigenvectors can be used as a model of the "baseline" state of the dynamic system. Using the generalized eigenvectors, new test conditions can then be tested to determine their "health".

## **PCA Transformation on a Single Test Condition**

To illustrate the mathematics of the proposed approach, vibration data from the triaxial accelerometer for Flight 1, file 4, Maneuver FFLS are used as an example. The  $n \times m$  input matrix for these data becomes  $\mathbf{X} = [XYZ]$ , where the columns X, Y, and Z correspond to the vibration data from the triaxial accelerometer for one test condition, synchronously averaged based on one revolution of the pinion gear (TSP data, n = 512). (X is the vertical direction, Y is the tangential direction, and Z is the radial direction.) It is assumed that the X, Y, Z data have been centered (mean is removed). For PCA, the m = 3 columns correspond to variables, and the n = 512 rows correspond to observations. PCA results in three output matrices, namely PC, SC, and LAT. The eigenvectors of the  $m \times m$  (m = 3) covariance matrix correspond to the columns of the  $m \times m$  (m = 3) PC matrix. The  $n \times m$  (m = 3)  $m \times m$ 

$$LAT = \begin{bmatrix} 365.1637 \\ 40.9655 \\ 14.8314 \end{bmatrix} \qquad PC = \begin{bmatrix} 0.1324 & -0.9142 & -0.3830 \\ 0.9680 & 0.2024 & -0.1486 \\ -0.2133 & 0.3510 & -0.9117 \end{bmatrix}$$

Algebraically, the principal components are linear combinations of the original variables X, Y, and Z (centered), which represent the selection of a new coordinate system after rotating the original coordinate system [6]. The first principal component, whose coefficients (eigenvectors) are indicated in the first column of the PC matrix, is the linear combination with the highest variance, described as 0.1324X + 0.9680Y - 0.2133Z (using centered variables X, Y, Z). This is computed as  $\mathbf{X} * PC$ , which is equivalent to the columns in the SC matrix. The coefficients imply that the leading principal component is weighted most by the original Y axis (0.9680 in the PC matrix), and about equally by the other two original axes. By contrast, the second principal component is weighted most by the X axis (-0.9142), and the third principal component by the Z axis (-0.9117). If the physical axes were set up perfectly for the original triaxial data, these weights would be 1.0, and the remaining weights would be equal to 0. The variance of the first principal component is equal to the first eigenvalue (the variance of the first column of the score matrix), computed as the first element in

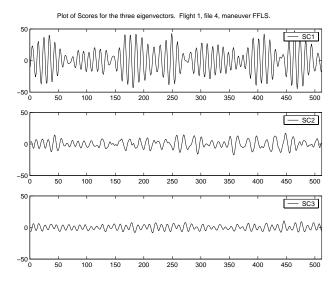
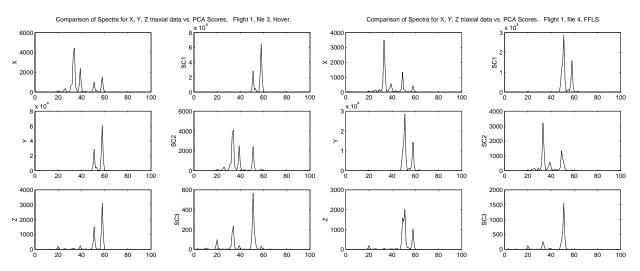


Figure 2: Scores: variation of the PCs over all observations (TSP data, flight 1, file 4, maneuver FFLS).

the *LAT* vector. The first principal component accounts for 86.75% of the total variance with an eigenvalue of  $\lambda_1 = 365.1637$ , whereas the second principal component accounts for 9.73% of the total variance with an eigenvalue equal to  $\lambda_2 = 40.9655$ .

Each column of the score matrix corresponds to the variation of the new eigenvectors (PC matrix) over the n = 512 observations. A plot of the scores is shown in Figure 2 for each of the eigenvectors. The first principal component represents the mode with the largest variance. Such plots can be used to monitor changes in each of the principal components [15].



**Figure 3:** Power spectra of triaxial vibration data in X, Y, and Z directions vs. power spectra of scores from principal components analysis. Flight 1, file 3, maneuver hover and flight 1, file 4, maneuver FFLS (TSP Data, Nfft=512).

To demonstrate how PCA is used to decorrelate the three vibrational directions and find an optimal direction for the triaxial accelerometer data, Figure 3 presents the comparison of the power spectra in the X,

Y, Z directions with the power spectra of the scores for the new "directions" described by the decorrelated principal components SC1, SC2, and SC3, for two of the maneuvers, Hover and FFLS, flight 1, pilot 1 (files 3 and 4). As can be observed from these comparative plots, the tangential direction Y is equivalent in frequency content to the first principal component scores (SC1) and the vertical direction X is equivalent to the second principal component scores (SC2). The results throughout the experiment show that for the TSP data, the triaxial accelerometer data are optimal in the sense that one of the directions corresponds to the direction of maximum variance defined by the first principal component.

## PCA Transformation on the Overall Experiment

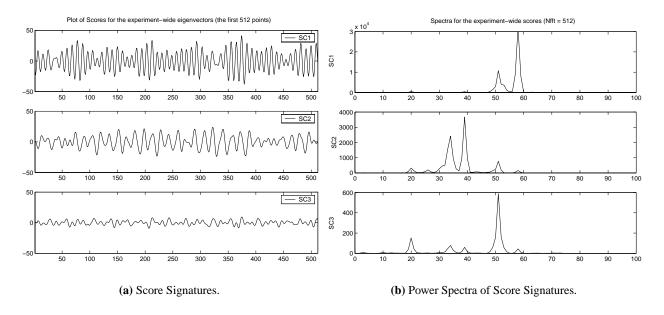
As the PCA transformation extracts the principal "modes" of vibration from the input data, it is hypothesized in this work that there will be similarities between the individual test conditions. If generalized modes of vibration exist, the eigenvectors should look similar, with different weights for each test condition indicating the changes due to the experimental factors (projection of each individual condition onto the experiment-wide eigenvectors.) To test this hypothesis, the PCA transformation is performed on an input matrix that includes all of the individual test conditions, concatenated into one large matrix. Each individual  $n \times m$  input matrix is  $\mathbf{X_i} = [X_i Y_i Z_i]$ , where the columns  $X_i$ ,  $Y_i$ , and  $Z_i$  correspond to the vibration data from the triaxial accelerometer for one test condition, synchronously averaged based on one revolution of the pinion gear (TSP data, n = 512, m = 3.) The overall input matrix  $\mathbf{X_{all}}$  has all of the test conditions including 22 files for each of the 8 flights, adding up to 176 files. The dimensionality of  $\mathbf{X_{all}}$  is  $N \times M$ , where  $N = 512 \times 22 \times 8 = 90112$  and M = 3 in this case. PCA (performed in Matlab) for the entire set of test conditions results in the following outputs:

$$LAT_{all} = \begin{bmatrix} 299.1607 \\ 59.4100 \\ 6.9852 \end{bmatrix} \qquad PC_{all} = \begin{bmatrix} -0.1853 & -0.9458 & -0.2667 \\ -0.9612 & 0.2310 & -0.1511 \\ 0.2045 & 0.2283 & -0.9519 \end{bmatrix}$$

As in the case of the individual test conditions, the coefficients in the  $PC_{all}$  matrix indicate that the first principal component (accounting for 81.94% of the total variance from the  $LAT_{all}$  vector) is weighted most by the original Y axis (-0.9612), the second principal component (accounting for 16.25% of the total variance) by the original X axis (-0.9458), and the third principal component by the Z axis (-0.9519). The scores for the experiment-wide input matrix are shown in Figure 4(a) for the first N = 512 points for comparison with the scores for the individual test conditions (see Figure 2). The power spectra corresponding to these scores are shown in Figure 4(b) for comparison with the power spectra for the individual cases discussed in Figure 3. The similarities between the individual egenvectors and the experiment-wide eigenvectors are apparent from this comparison. Individual test conditions can be analyzed using the eigenvector models generated from the entire set of experiments.

For example, a new test condition can be analyzed by projecting the vibration data onto the set of generalized eigenvectors (representing the overall baseline state of the flight, including all vibrational modes). The corresponding score matrices will determine whether the new test condition belongs to the general baseline state or whether it deviates from it, implying a potential failure or defect.

To investigate the directionality of the individual test conditions compared to the principal directions dictated by the experiment-wide eigenvectors, the angles of the principal components are compared next. The main two angles that the first principal component for the experiment-wide case makes with the original X, Y, Z axes (from  $\mathbf{X}_{all}$ ) are computed, followed by the angle of the first principal component computed from each of the individual test conditions (from each  $\mathbf{X}_i$ ). Figure 5 shows a plot of the difference between the individual angles and the experiment-wide angle (using the first angle  $\theta$ , see [13] for details) for the first principal component.



**Figure 4:** Experiment-wide PCA Score Vectors (the first N = 512 points are shown for comparison with individual case; power spectra have been computed using Nfft = 512.)

The changes in this angle indicate changes in the directionality of the first principal component, which indicates the "optimum" direction of vibration (i.e., the mathematical direction with maximum variance.) The x-axis represents the flights: test case 1-22 corresponds to flight 1, cases 23-44 corresponds to flight 2, etc. As noted, two separate patterns in directionality are identified: flights  $\{1,3,5,7\}$  and flights  $\{2,4,6,8\}$  are separated into two distinct patterns, representing two different sets of maneuvers [13]. Monitoring of directional changes of the principal components is hence likely to present a good means to distinguish between these maneuvers.

#### **Conclusions and Future Work:**

This paper presents the foundations of an approach to use triaxial vibration measurements to determine the health and condition of a dynamic system. Flight data collected using an OH58C helicopter are used to demonstrate the mechanics of the proposed approach. Vibration data are collected using a triaxial accelerometer and analyzed in each individual direction for potential changes and variations. The data in the three directions are then combined using Principal Components Analysis (PCA). The PCA transformation provides a means to analyze and monitor all three directions in a combined form. More specifically, a "principal direction" is computed as a linear combination of the three axes from the vibration measurements, maximized with respect to the total variance in the data.

In this paper, the proposed transformation is applied to the entire set of test conditions to derive "experiment-wide" eigenvectors, which provide a model of the baseline (healthy) state of the dynamic system under study, capturing the dominant vibrational modes. Each new test condition is then projected onto these generalized eigenvectors to determine their state and condition. Deviations can be observed in the projection of new states onto the generalized eigenvectors, indicating potential anomalies and developing failures in the system. Finally, the changes in the directionality of the principal axis of vibration are monitored using the angles

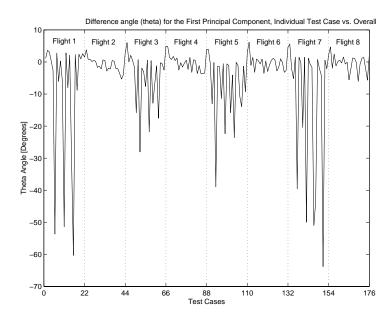


Figure 5: Difference of angles between individual and experiment-wide PCA eigenvectors, 1st PC.

the transformed axes make with the original axes of vibration. The example in this paper shows a clustering by maneuvers, potentially presenting a useful metric for vibration monitoring.

Further research is necessary to investigate the value of projecting new (faulty) test conditions onto the generalized eigenvectors for failure detection. The preliminary results, presented in this paper, demonstrate the potential value of using triaxial vibration recordings, in conjunction with the proposed PCA-based approach, to monitor changes in the vibrational signatures during flight. The observed differences in the PCA output variables (eigenvalues, eigenvectors, or the derived rotation angles) need to be studied further to assure that sufficient statistical sampling is provided and to understand their sampling distributions.

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